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ERRATUM TO: " NORMALITY AND COUNTABLE PARACOMPACTNESS OF HYPERSPACES OF ORDINALS", TOPOLOGY AND ITS APPLICATIONS 154 (2007) 358-362

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Abstract. We correct the proof of Theorem 8 in " Normality and countable paracompactness of hyperspaces of ordinals", Topology and its Applications 154 (2007) 358-362.

Prof. Y. Hirata kindly informed to the author that there is a gap in the proof of Theorem 8 in [2], moreover he gave suggestions to correct it. Although Theorem 8 itself remains true, we now improve its proof. All other results and proofs are true.

Theorem 8. *If κ is a regular uncountable cardinal, then $\mathcal{K}(\kappa)$ is normal.*

Proof. Let $\mathcal{F}$ and $\mathcal{H}$ be disjoint closed sets in $\mathcal{K}(\kappa)$. Let M_0 be an elementary submodel of $H(\theta)$, where θ is large enough, such that $\mathcal{F}, \mathcal{H}, \kappa \in M_0$ and $|M_0| < \kappa$. For elementary submodels, the readers should refer to [1, 3]. Assume that elementary submodels $M_0, ..., M_{n-1}$ of $H(\theta)$ with $M_0 \subset ... \subset M_{n-1}$ and $|M_{n-1}| < \kappa$ are defined. Let M_n be an elementary submodel of $H(\theta)$ satisfying $M_{n-1} \cup \bigcup(M_{n-1} \cap \kappa) \subset M_n$ and $|M_n| < \kappa$. Then the union $M = \bigcup_{n\in\omega} M_n$ is also an elementary submodel of $H(\theta)$ and satisfies $\mathcal{F}, \mathcal{H}, \kappa \in M$, $|M| < \kappa$ and $\kappa \cap M$ is an ordinal. Let $\gamma = \kappa \cap M < \kappa$.

Claim 1. If $F \in \mathcal{K}(\kappa) \cap M$, then $\max F < \gamma$.

Proof. Since F is a compact subset of κ, $\max F$ exists and is an element of κ. On the other hand $\max F$ is deteremined by F and $F \in M$, by elementarity, we have $\max F \in M$. Therefore $\max F \in \kappa \cap M = \gamma$.

Observe that by Claim 1, $\mathcal{F} \cap M$ and $\mathcal{H} \cap M$ are subsets of the compact space $\mathcal{K}([0, \gamma]) = 2^{[0,\gamma]} \subset \mathcal{K}(\kappa)$. Let $\mathcal{F}_M = \mathrm{Cl}_{\mathcal{K}([0,\gamma])}(\mathcal{F}\cap M)$ and $\mathcal{H}_M = \mathrm{Cl}_{\mathcal{K}([0,\gamma])}(\mathcal{H}\cap M)$. Then $\mathcal{F}_M \cap \mathcal{H}_M \subset \mathcal{F} \cap \mathcal{H} = \emptyset$. Since $\mathcal{K}([0, \gamma])$ is normal in fact compact T_2, there are disjoint open sets $U_\mathcal{F}$ and $U_\mathcal{H}$ separating $\mathcal{F}_M$ and $\mathcal{H}_M$ respectively. For each $K \in \mathcal{F}_M \cup \mathcal{H}_M$, fix a finte collection $\mathcal{V}_K$ of open sets in $[0, \gamma]$ such that

- if $K \in \mathcal{F}_M$, then $K \in \langle\mathcal{V}_K\rangle \subset U_\mathcal{F}$,
- if $K \in \mathcal{H}_M$, then $K \in \langle\mathcal{V}_K\rangle \subset U_\mathcal{H}$,

Applying Lemma 7 in [2] to $\gamma + 1 = [0, \gamma]$, for each $K \in \mathcal{F}_M \cup \mathcal{H}_M$, by $K \in \langle\mathcal{V}_K\rangle$ we can find two decreasing sequences $\{\alpha^K_i : i \leq n_K\}$ and $\{\beta^K_i : i < n_K\}$ in $[0, \gamma]$ satisfying

(1) $\alpha^K_0 = \max K$, $\{\alpha^K_i : i < n_K\} \subset K$.
(2) $\alpha^K_{i+1} \leq \beta^K_i < \alpha^K_i$ for each $i < n_K$, where $\alpha^K_{n_K} = -1$.

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(3) $K \in \langle\{(\beta_i^K, \alpha_i^K] : i < n_K\}\rangle \subset \langle\mathcal{V}_K\rangle$.

We may assume $\mathcal{V}_K = \{(\beta_i^K, \alpha_i^K] : i < n_K\}$ for each $K \in \mathcal{F}_M \cup \mathcal{H}_M$.

Since $\mathcal{K}([0,\gamma])$ is compact, one can find two finite sets $\mathcal{F}'$ and $\mathcal{H}'$ of $\mathcal{F}_M$ and $\mathcal{H}_M$ respectively such that $\mathcal{F}_M \subset \bigcup_{K\in\mathcal{F}'}\langle\mathcal{V}_K\rangle$ and $\mathcal{H}_M \subset \bigcup_{K\in\mathcal{H}'}\langle\mathcal{V}_K\rangle$. Remark that by (2), all α_i^K's ($1 \le i \le n_K$) and β_i^K's ($0 \le i < n_K$) belong to M and that α_0^K belongs to M iff $\alpha_0^K < \gamma$.

Now for each $K \in \mathcal{F}' \cup \mathcal{H}'$ and $i < n_K$, let

$$W_i^K = \begin{cases} (\beta_i^K, \kappa) & \text{if } i = 0 \text{ and } \alpha_i^K = \gamma, \\ (\beta_i^K, \alpha_i^K] & \text{otherwise.} \end{cases}$$

Then by the remark above and $\kappa \in M$, for each $K \in \mathcal{F}' \cup \mathcal{H}'$ and $i < n_K$, we have $W_i^K \in M$ therefore $\mathcal{W}_K = \{W_i^K : i < n_K\}$ is a pairwise disjoint collection of intervals in κ that belongs to M. Now we consider the open sets $W_\mathcal{F} = \bigcup_{K\in\mathcal{F}'}\langle\mathcal{W}_K\rangle$ and $W_\mathcal{H} = \bigcup_{K\in\mathcal{H}'}\langle\mathcal{W}_K\rangle$ in $\mathcal{K}(\kappa)$. Since $W_\mathcal{F}$ and $W_\mathcal{H}$ are definable from $\mathcal{W}_K$'s moreover $\mathcal{F}'$ and $\mathcal{H}'$ are finite, we have $W_\mathcal{F}, W_\mathcal{H} \in M$. It suffices to see the following two claims.

Claim 2. $W_\mathcal{F} \cap W_\mathcal{H} = \emptyset$

Proof. Assume $W_\mathcal{F} \cap W_\mathcal{H} \neq \emptyset$, then there are $K(\mathcal{F}) \in \mathcal{F}'$ and $K(\mathcal{H}) \in \mathcal{H}'$ such that $\langle\mathcal{W}_{K(\mathcal{F})}\rangle \cap \langle\mathcal{W}_{K(\mathcal{H})}\rangle \neq \emptyset$. By $\mathcal{W}_{K(\mathcal{F})}, \mathcal{W}_{K(\mathcal{H})} \in M$ and elementarity, we have $M \models \langle\mathcal{W}_{K(\mathcal{F})}\rangle \cap \langle\mathcal{W}_{K(\mathcal{H})}\rangle \neq \emptyset$, thus there is $L \in \langle\mathcal{W}_{K(\mathcal{F})}\rangle \cap \langle\mathcal{W}_{K(\mathcal{H})}\rangle \cap M$. Note by Claim 1 that $\max L < \gamma$ holds therefore we have $L \in \mathcal{K}([0,\gamma])$.

Now using the definition of W_i^K's, it is straightforward to see that in $\mathcal{K}([0,\gamma])$, $L \in \langle\mathcal{V}_{K(\mathcal{F})}\rangle \cap \langle\mathcal{V}_{K(\mathcal{H})}\rangle \subset U_\mathcal{F} \cap U_\mathcal{H}$ holds, a contradiction.

Claim 3. $\mathcal{F} \subset W_\mathcal{F}$ and $\mathcal{H} \subset W_\mathcal{H}$.

Proof. Assume $\mathcal{F} \setminus W_\mathcal{F} \neq \emptyset$. By elementarity and $\mathcal{F}, W_\mathcal{F} \in M$, there is $L \in (\mathcal{F} \setminus W_\mathcal{F}) \cap M$. It follows from $L \in \mathcal{F} \cap M \subset \bigcup_{K\in\mathcal{F}'}\langle\mathcal{V}_K\rangle$ that $L \in \langle\mathcal{V}_K\rangle$ for some $K \in \mathcal{F}'$.

Now by the definition of W_i^K's, we have $L \in \langle\mathcal{W}_K\rangle \subset W_\mathcal{F}$ in $\mathcal{K}(\kappa)$, a contradiction. We see $\mathcal{F} \subset W_\mathcal{F}$, the rest is similar.

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